IS 6200-4 (2008): Statistical Tests of Significance, Part 4: Non-parametric Tests [MSD 3: Statistical Methods for Quality and Reliability]

BLANK PAGE

भारतीय मानक

सार्थकता के लिए सांख्यिकीय परीक्षण

भाग 4 गैर-पैरामीटरी परीक्षण

(पहला पुनरीक्षण)

Indian Standard

STATISTICAL TESTS OF SIGNIFICANCE

PART 4 NON-PARAMETRIC TESTS

(First Revision)

ICS 03.120.30

BUREAU OF INDIAN STANDARDS
MANAK BHAVAN, 9 BAHADUR SHAH ZAFAR MARG
NEW DELHI 110002

July 2008

Price Group 9

Statistical Methods for Quality and Reliability Sectional Committee, MSD 3

FOREWORD

This Indian Standard (Part 4) (First Revision) was adopted by the Bureau of Indian Standards, after the draft finalized by the Statistical Methods for Quality and Reliability Sectional Committee had been approved by the Management and Systems Division Council.

This standard was first issued in the year 1983 and was reviewed in the light of statistical changes required in the present context as also modification of some of the concepts, presentation in line with other Indian Standards on the subject.

Statistical tests of significance are important tools in industrial experimentation and decision making. These tests may broadly be classified into two categories, namely, parametric tests and non-parametric tests. A parametric test is a test whose model specifies certain assumptions about the parameters of the population from which the sample is drawn. The statistical tests described in Part 1 of this standard are parametric tests as these tests are concerned with the hypothesis about the parameters of the population. For example, in the case of t-test and F-test, it is assumed that the variances of the two populations are the same. Further, it is also assumed that the samples are drawn from a normal population. So the meaningfulness of the results of a parametric test depends upon the validity of these assumptions. It is, therefore, necessary to verify these assumptions before applying a parametric test. But these assumptions are not ordinarily tested and are assumed to hold good. These assumptions, therefore, restrict the wider applicability of these tests.

A number of statistical tests of significance are available which do not make the assumption of normality of the parent population. These tests are known as non-parametric or distribution-free tests and are based on ranking or ordering of observations or on number of observations exceeding or falling short of a given value.

This standard describes some of the important non-parametric tests for testing whether the two samples are drawn from the same population. The two samples may be independent or related to each other. Some of the tests described in this standard are applicable when samples are drawn from related populations, while the others are applicable to samples from independent populations. Kolmogorov-Smirnov one-sample test is also included for testing whether a sample has been drawn from a given population.

Indian Standard IS 6273 (Part 3) : 1983 'Guide for sensory evaluation of foods: Part 3 Statistical analysis of data' also gives the applications of some of the non-parametric tests in the analysis of data arising from sensory evaluation experiments. For further details, reference may be made to this standard.

The statistical tests described in Part 1 of this standard are normal, t and D tests. χ^2-test and tests for normality are covered in Parts 2 and 3 respectively.

In reporting the results of a test or analysis made in accordance with this standard, if the final value, observed or calculated, is to be rounded off, it shall be done in accordance with IS 2 : 1960 'Rules for rounding off numerical values (*revised*)'.

Indian Standard

STATISTICAL TESTS OF SIGNIFICANCE

PART 4 NON-PARAMETRIC TESTS

(First Revision)

1 SCOPE

1.1 This standard (Part 4) lays down the following tests of significance:

a) Kolmogorov-Smirnov one-sample test,
b) Kolmogorov-Smirnov two-sample test,
c) Sign test,
d) Wilcoxon matched-pairs sign-rank test,
e) Median test,
f) Mann-Whitney U test, and
g) Wald-Wolfowitz run test.

1.2 For these tests, procedures to deal with small samples and large samples have been explained separately in this standard. The power efficiency of each test has also been indicated.

2 REFERENCES

The following standards contain provisions, which through reference in this text, constitute provisions of this standard. At the time of publication, the editions indicated were valid. All standards are subject to revision and parties to agreements based on this standard are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below:

IS No.	*Title*
6200 (Part 2): 2004	Statistical tests of significance : Part 2 χ^2-test (*second revision*)
7920	Statistical vocabulary and symbols:
(Part 1) : 1994	Probability and general statistical terms (*second revision*)
(Part 2) : 1994	Statistical quality control (*second revision*)

3 TERMINOLOGY

For the purpose of this standard, the definitions given in IS 7920 (Part 1) and IS 7920 (Part 2) shall apply.

4 ADVANTAGES AND DISADVANTAGES OF NON-PARAMETRIC TESTS

4.1 The non-parametric tests have the following advantages:

a) These tests can be used even if the assumption of normality of the parent population is unrealistic. (The claim that 'probability statements from N-P tests are exact probabilities regardless of the shape of population distribution' is not true; for example, the power of sign test depends on the form of the population of differences.)
b) For sample size less than or equal to 6, there is no alternative but to use a non-parametric statistical tests unless the nature of the population distribution is known exactly.
c) There are suitable non-parametric statistical tests in treating samples made up of observations from several different populations. None of the parametric test can handle such data without requiring to make unrealistic assumptions.
d) Non-parametric statistical tests are available to treat data which are inherently in ranks as well as the data whose seemingly numerical scores have the strength of ranks, for example, where one is able to select one of the two characteristics in preference to the other. But this type of data cannot be treated by parametric method unless some realistic assumption is made about the underlying distribution.
e) Non-parametric tests are much easier to learn, calculate and apply as compared to parametric tests.
f) Non-parametric methods are available to treat the data which are simply classificatory. No parametric techniques can be applied to such data.

4.2 The non-parametric tests have some disadvantages also. If all the assumptions of the relevant parametric statistical model are met, then non-parametric tests are less sensitive. The degree of sensitivity is expressed by the power-efficiency of the non-parametric test.

5 BASIC CONCEPTS

5.1 Statistical tests of significance are important tools in decision-making. They are extremely useful in finding out whether, in the case of one population, the mean value differs significantly from certain specified value or whether. in the case of two populations, the mean values differ significantly from each other. Thus, it may be desirable to find out whether a new germicide is more effective in treating a certain type of infection than a standard germicide, whether a new method of sealing light bulbs will increase their life or whether one method of preserving foods is better than another insofar as the retention of vitamins is concerned. In such cases, it would be necessary to examine whether the mean values can be deemed as same or different. There may also be cases where it may be worthwhile to find out whether one inspector is more consistent than another or whether a new source of raw material has resulted in a change in the variability of the output, or whether the temperature of the bath in which the cocoons are cooked affects the uniformity of the quality of silk. In these cases it will be necessary to determine whether the variances are the same or not.

5.2 Formulation of Hypotheses

For taking a decision using statistical tests of significance, the first step is to form the hypotheses, namely, Null Hypothesis (H_0) and Alternative Hypothesis (H_1).

5.2.1 *Null Hypothesis (H_0)*

The procedure commonly used is to first set up a null hypothesis regarding equivalence (no difference). The question, on which the decision is called for, by applying the tests of significance, is translated in terms of null hypothesis in such a way that this null hypothesis would likely to be rejected if there is enough evidence against it as seen from the data in the sample. For example, a null hypothesis will be that the data follow a normal distribution.

5.2.2 *Alternative Hypothesis (H_1)*

Alternative hypothesis is a hypothesis that will be preferred in case the null hypothesis is not true.

5.3 Level of Significance

5.3.1 There are two kinds of errors involved in taking the decision based on the tests of significance, namely:

a) *Type I Error* — Error in deciding that a significant difference exists when there is no real difference.

b) *Type II Error* — Error in deciding that no difference exists when there is a real difference.

5.3.2 Type I error and Type II errors are also called Error of the first kind and Error of the second kind respectively. This process of decision making is described in the table given below:

	H_0 True	*H_1 True*
Reject H_0	Type I error	Correct decision
Accept H_0	Correct decision	Type II error

5.3.3 Based on the distribution of test statistics used, it is possible to work out the probability of committing Type I error. The probability of committing Type I error is called level of significance (α). The probability of committing Type II error is called level of significance (β). It is not possible to minimize both these probabilities (risk) at the same time. Hence, assigning to it a chosen level of probability controls one of the risks, usually of the first kind. Generally the value for level of significance is chosen as 0.05 or 0.01, that is, 5 percent or 1 percent. This implies confidence level of 95 percent or 99 percent respectively.

5.4 The decision-making procedure involves the comparison of the calculated value of the statistic with the tabulated value. If the calculated value is greater than or equal to the tabulated value of the statistic, then H_0 is rejected, thereby accepting H_1; otherwise H_0 is not rejected. For practical purpose, H_0 not rejected is taken as if it is accepted.

5.5 *p*-Value Approach for Statistical Tests of Significance

p-Value approach for statistical tests of significance can also be used for this purpose. For performing any test of significance, the probability *p* of the test-statistic assuming, under the null hypothesis, the observed value and more likely values favouring the alternative hypothesis is calculated. This *p*-value is given alongside the observed value of the test-statistic in statistical packages used for performing such tests. If the calculated *p*-value is less than the chosen level of significance α, the null hypothesis is rejected, otherwise it is accepted. The advantage of this approach is that here there is no need to look for critical values of the test-statistic in Statistical Tables.

6 KOLMOGOROV-SMIRNOV ONE-SAMPLE TEST

6.1 The Kolmogorov-Smirnov one-sample test is a test of goodness of fit, that is, it is concerned with the degree of agreement between the distribution of a set of sample values and some specified theoretical distribution. It determines whether the sample values can reasonably be thought to have come from a population having a theoretical distribution with known parameters.

6.2 The test is accomplished by finding the theoretical cumulative frequency distribution which would be expected under the null hypothesis [$F(X)$] and comparing it with the observed cumulative frequency distribution [$S_n(X)$]. Under the null hypothesis that the sample has been drawn from the specified theoretical distribution, it is expected that for every value of X, $S_n(X)$ should be fairly close to $F(X)$, that is, the differences between the theoretical and observed distribution should be small and within the limits of random errors. The point at which these two distributions, theoretical and observed, show the maximum deviation is determined.

$$\text{Let } D = \text{Maximum } |F(X) - S_n(X)|$$

6.3 This value of D is calculated and compared with the critical value given in Annex A for desired level of significance. The null hypothesis is rejected if the calculated value of D is greater than the critical value; otherwise not. Alternatively, if the calculated p-value of the test-statistic assuming, under H_0, the observed value and more likely values favouring H_1, is less than the chosen level of significance α, H_0 is rejected.

6.4 If the sample size (n) is more than 35, the critical value of D is $1.36/\sqrt{n}$ for 5 percent level of significance and $1.63/\sqrt{n}$ for 1 percent level of significance.

6.5 Example 1

The mean and standard deviation of 150 observations of leakage current taken on 150 electric irons are 37.6 µA and 12.30 µA respectively. These values are given in form of frequency distribution in Table 1. It has to be tested whether the data follow normal distribution.

Table 1 Frequency Distribution of Leakage Current

Class-Interval (1)	Frequency (2)
10.5-15.5	6
15.5-20.5	8
20.5-25.5	12
25.5-30.5	16
30.5-35.5	21
35.5-40.5	30
40.5-45.5	18
45.5-50.5	15
50.5-55.5	10
55.5-60.5	9
60.5-65.5	5
Total	150

6.5.1 *Null Hypothesis (H_0) and Alternative Hypothesis (H_1)*

The null hypothesis is that the data follow the normal distribution against an alternative hypothesis (H_1) that the data do not follow the normal distribution.

6.5.2 The area to the left of upper limit of each class-interval, as shown in Table 2, is calculated with the help of standard normal probability tables.

6.5.3 The critical value of D at 5 percent level of significance is $1.36/\sqrt{150} = 0.111\ 0$. Since the calculated value of D is less than the critical value, the null hypothesis that the data follow normal distribution is not rejected. The 2-tailed p-value obtained by using SPSS package comes out as $0.064\ 2 > 0.05$, so this conclusion is further confirmed.

6.6 Power Efficiency

The Kolmogorov-Smirnov one-sample test treats individual observations separately and thus, unlike χ^2-test for one sample, need not lose information through the combining of categories. When expected frequencies of some classes are less than 5, adjacent categories shall be combined before χ^2 may properly be computed. So the χ^2-test is less powerful than the Kolmogorov-Smirnov test. Moreover, for very small samples the χ^2-test is not applicable at all, but the Kolmogorov-Smirnov test is. These facts suggest that the Kolmogorov-Smirnov test may, in all cases, be more powerful than its alternative, the χ^2-test.

7 KOLMOGOROV-SMIRNOV TWO-SAMPLE TEST

7.1 The Kolmogorov-Smirnov two-sample test is used to test whether two independent samples have been drawn from the same population, that is, populations having the same distribution. The two-sided test is sensitive to any kind of difference in the distributions from which the two samples are drawn, that is, differences in location, in dispersion, in skewness, etc. The one-sided test is used to test whether or not the values of the population from which one of the samples is drawn is stochastically larger than the values of the population from which the other sample is drawn.

7.2 Like the Kolmogorov-Smirnov one-sample test, this two-sample test is concerned with the agreement between two cumulative distributions. The one-sample test is concerned with the agreement between the distribution of a set of sample values and some specified theoretical distribution. The two-sample test is concerned with the agreement between two sets of sample values.

7.3 If the two samples have in fact been drawn from the same population distribution, then the cumulative distributions of both samples may be expected to be fairly close to each other in as much as they both should show only random deviations from the population distribution. If the two sample cumulative distributions are too far apart at any point, this will suggest that the samples have come from different populations. Thus a

Table 2 Area Under Normal Curve to Left of Upper Limits of Class Intervals
(*Clause* 6.5.2)

Upper Limit of Class Interval (X)	$Z = \frac{X - 37.6}{12.3}$	$F(X)$ = Area to the Left of Z	Cumulative Frequency (Less Than X)	$S_n(X) = \frac{\text{col 4}}{150}$	$\lvert F(X) - S_n(X) \rvert$ [col 3 – col 5]
(1)	(2)	(3)	(4)	(5)	(6)
15.5	–1.80 0	0.035 9	6	0.040 0	0.004 1
20.5	–1.39	0.082 3	14	0.093 3	0.011 0
25.5	–0.98	0.163 5	26	0.173 3	0.009 8
30.5	–0.58	0.281 0	42	0.280 0	0.001 0
35.5	–0.17	0.432 5	63	0.420 0	0.012 5
40.5	0.24	0.594 8	93	0.620 0	0.025 2
45.5	0.64	0.738 9	111	0.740 0	0.001 1
50.5	1.05	0.853 1	126	0.840 0	0.013 1
55.5	1.46	0.927 9	136	0.906 7	0.021 2
60.5	1.86	0.968 6	145	0.966 7	0.001 9
65.5	2.27	0.988 4	150	1.000 0	0.011 6

Therefore D = Maximum $\lvert F(X) - S_n(X) \rvert$ = 0.025 2.

large enough deviation between the two sample cumulative distributions is evidence for rejecting the null hypothesis.

7.4 To apply the Kolmogorov-Smirnov two-sample test, a cumulative frequency distribution is obtained for each sample of observations, using the same intervals for both distributions. From the cumulative frequencies for both the samples, the cumulative step function values are calculated. Corresponding to each interval, then cumulative step function of one sample is subtracted from the other. The test focuses on the largest of these observed deviations.

7.5 Let $S_{n1}(X)$ = the observed cumulative step function of one of the samples, that is, $S_{n1}(X) = K/n_1$ where K is the number of observations equal to or less than X. Let $S_{n2}(X)$ = the observed cumulative step function of the other sample, that is, $S_{n2}(X) = K/n_2$, then:

D = maximum $[S_{n1}(X) - S_{n2}(X)]$ is taken as test criteria for a one-sided test, and

D = maximum $\lvert S_{n1}(X) - S_{n2}(X) \rvert$ for a two-sided test.

7.6 This value of D is calculated and compared with the critical value for desired level of significance. The null hypothesis is rejected if the critical value is greater than the calculated value, otherwise not. Alternatively, if the calculated p-value of the test-statistic assuming, under H_0, the observed value and more likely values favouring H_1, is less than the chosen level of significance α, H_0 is rejected.

7.7 Small Samples

When $n_1 = n_2$ and both n_1 and n_2 are 40 or less, Annex B may be used for testing the null hypothesis. This Annex gives various values of K_D, which is defined as the numerator of the largest difference between the two cumulative step functions, that is, the numerator of D. To read Annex B, one must know the value of n (which in this case is the value of $n_1 = n_2$) and the desired level of significance. Observe also whether alternative hypothesis (H_1) calls for a one-sided or a two-sided test. With this information, one may determine the significance of the observed data.

7.8 Example 2

Two machines were used for the production of cylinders in a workshop. A random sample of 10 cylinders from each of the two machines was selected to determine its diameter. The values of the diameter are given in Table 3.

Table 3 Diameters of Cylinders, mm

Machine I	Machine II
122	123
118	126
126	121
124	124
127	125
119	121
121	125
125	128
121	123
120	124

It has to be tested whether the two machines are producing the cylinders of same diameter.

7.8.1 *Null Hypothesis (H_0) and Alternative Hypothesis (H_1)*

The null hypothesis is that the two machines are producing the cylinders of same diameter against the alternative hypothesis (H_1) that the two machines are not producing the cylinders of same diameter.

7.8.2 The cumulative frequency distributions of both the samples are given in Table 4.

7.8.3 The critical value of K_D from Annex B at 5 percent level of significance is 7. Since the calculated value is less than the critical value, the null hypothesis that the two machines are producing the cylinders of same diameter is not rejected.

7.9 Large Samples

7.9.1 *Two-Sided Test*

When both n_1 and n_2 are larger than 40, the critical values for the Kolmogorov–Smirnov two-sample test, at desired level of significance are calculated by the following relation:

$1.36\sqrt{\frac{n_1+n_2}{n_1n_2}}$ for 5 percent level of significance,

and

$1.63\sqrt{\frac{n_1+n_2}{n_1n_2}}$ for 1 percent level of significance.

The value of D as calculated in **7.5** is compared with this critical value. The null hypothesis is rejected if the critical value is less than the calculated value of D, otherwise not. Alternatively, if the calculated p-value of the test-statistic assuming, under H_0, the observed value and more likely values favouring H_1, is less than the chosen level of significance α, H_0 is rejected.

7.9.2 *One-Sided Test*

When n_1 and n_2 are large, and regardless of whether or not $n_1 = n_2$, the value of D for one-sided test is calculated as given in **7.5**, that is, by using the following relation:

$$D = \text{Maximum } [Sn_1(X) - Sn_2(X)]$$

It has been shown that

$$\chi^2 = 4D^2 \frac{n_1n_2}{n_1+n_2}$$

is approximately distributed as chi-square (χ^2) with two degrees of freedom. Thus, calculating the value of χ^2 from the above relation and comparing it with the tabulated value of χ^2 for two degrees of freedom at desired level of significance, one may determine whether the null hypothesis is rejected, or not. Alternatively, if the calculated p-value of the test-statistic assuming, under H_0, the observed value and more likely values favouring H_1, is less than the chosen level of significance α, H_0 is rejected.

8 SIGN TEST

8.1 For testing equality of means of two correlated populations (X, Y) against one-sided or two-sided alternatives, the parametric paired t-test cannot be used if the normality assumption of the underlying bivariate population is unrealistic. In this case, one can conveniently use paired-sample sign test. This test can be used even if instead of actual paired measurements (x_i, y_i), only data on whether x_i is greater than or less than y_i are available.

Table 4 Cumulative Frequency Distributions
(*Clause* 7.8.2)

Class-Interval	Frequency First Sample		Frequency Second Sample		$S_1(x)$	$S_2(x)$	D
	Frequency	Cumulative Frequency	Frequency	Cumulative Frequency	$\frac{\text{col 3}}{10}$	$\frac{\text{col 5}}{10}$	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
117.5-118.5	1	1	0	0	0.1	0.0	0.1
118.5-119.5	1	2	0	0	0.2	0.0	0.2
119.5-120:5	1	3	0	0	0.3	0.0	0.3
120.5-121.5	2	5	2	2	0.5	0.2	0.3
121.5-122.5	0	5	1	3	0.5	0.3	0.2
122.5-123.5	1	6	2	5	0.6	0.5	0.1
123.5-124.5	1	7	2	7	0.7	0.7	0.0
124.5-125.5	1	8	1	8	0.8	0.8	0.0
125.5-126.5	1	9	1	9	0.9	0.9	0.0
126.5-127.5	1	10	0	9	1.0	0.9	0.1
127.5-128.5	0	10	1	10	1.0	1.0	0.0

Therefore, D = maximum $|S_1(x) - S_2(x)|$ = 0.3 = 3/10.
K_D = numerator of D = 3.

NOTE — D is expressed as fraction k/n where n, is the sample size.

8.2 Null Hypothesis and Alternative Hypothesis

Here the null hypothesis is $H_0 : u_e = 0$ against one of the alternatives $H_1 : u_e > 0$, or $H_2 : u_e < 0$ or $H_3 : u_e \neq 0$, where u_e is the median of the population of differences $D = X - Y$.

8.3 Test-Statistic

For each of n pairs of sample observations (x_i, y_i), $d_i = x_i - y_i$, $i = 1, 2, \ldots, n$ is calculated and only the sign (plus or minus) of each such difference is noted, ignoring the magnitudes of differences altogether. Differences exactly equal to zero are ignored, and n reduced accordingly.

Under the null hypothesis, it is expected that numbers of plus and minus signs should be equal. Therefore, the null hypothesis should be rejected if one kind of sign occurs predominantly in larger number than the other.

8.4 Small Samples

This method shall be employed when the number of pairs (n) in the sample is less than or equal to 25. From the data, the number of fewer signs (say x) is calculated. This calculated value of x is then compared with critical value of x corresponding to sample size n and desired level of significance. The critical values for this purpose are given in Annex C. Depending upon the alternative hypothesis which may determine whether the test is two-sided or one-sided, the appropriate critical value may be selected. If the critical value is greater than the calculated value, the null hypothesis is rejected; otherwise not. Alternatively, if the calculated p-value of the test-statistic assuming, under H_0, the observed value and more likely values favouring H_1, is less than the chosen level of significance α, H_0 is rejected.

8.5 Example 3

The percentage yields (X_A and X_B) of a medicinal product in two chemical processes are given in Table 5. It has to be tested whether the data provide evidence of difference between the two processes.

Table 5 Percentage Yields of Medicinal Product

Batch No.	X_A	X_B
1	60.1	63.9
2	57.0	60.3
3	58.6	58.5
4	58.8	61.3
5	60.2	59.7
6	58.0	61.0
7	59.2	60.8
8	60.1	60.2

8.5.1 *Null Hypothesis (H_0) and Alternative Hypothesis (H_1)*

The data does not provide the evidence of a difference between the two processes against an alternative hypothesis (H_1) that there is a difference between the two processes.

8.5.2 The sign of difference ($X_A - X_B$) is given below:

Batch No.	1	2	3	4	5	6	7	8
Sign of ($X_A - X_B$)	–	–	+	–	+	–	–	–

Number of '+ve' signs = 2

Number of ' –ve' signs = 6

Therefore, x = Number of fewer signs = 2

From Annex C, the critical value of x for two-sided test for n = 8 and 5 percent level of significance is zero. Since the critical value is less than the calculated value, the null hypothesis is not rejected.

Since the 2-tailed p-value comes out as 0.289 1 > 0.05, the decision is upheld.

8.6 Large Samples

When the sample size is more than 25, the normal assumption may be used. The distribution of x (the number of either positive or negative signs) is a binomial with mean 'np' and variance '$np(1-p)$' where n is the sample size and p is the probability of occurrence of positive or negative sign. Thus the standardized normal test is given by:

$$Z = \frac{|x - np|}{\sqrt{np(1-p)}}$$

$$Z = \frac{|2x - n|}{\sqrt{n}}, \text{ under the null hypothesis of } p = ½$$

8.6.1 The value of Z is calculated and compared with critical value for 5 percent level or 1 percent level of significance for a one-sided or a two-sided test, as given below:

Significance Level(α)	*One-Sided Test*	*Two-Sided Test*
0.05	1.645	1.960
0.01	2.326	2.576

The null hypothesis is rejected if the calculated value is greater than the critical value, otherwise not rejected. Alternatively, if the calculated p-value of the test-statistic assuming, under H_0, the observed value and more likely values favouring H_1, is less than the chosen level of significance α, H_0 is rejected.

8.7 Example 4

In the export of iron ore, the iron content of the ore is determined at the loading port by using the sampling method, prevalent in the exporting country. Again, the same shipment is sampled by the importing country by their own method for the determination of iron content. The results of 40 shipments of iron ore exports from India to an overseas country are given in Table 6. It is intended to find whether the two methods of sampling adopted by the exporting and importing countries (in the estimation of iron content) are significantly different from each other. It is assumed that there is no change in quality of iron ore during transit.

Table 6 Iron Content of Iron Ore

Shipments	Iron Content at Loading Port	Iron Content at Unloading Port	Sign of Difference
(1)	(2)	(3)	(4)
1	64.74	65.11	–
2	64.53	65.71	–
3	64.28	65.16	–
4	64.97	65.44	–
5	63.62	63.73	–
6	65.62	65.16	+
7	64.46	64.46	0
8	65.03	64.25	+
9	64.80	66.22	–
10	64.53	65.17	–
11	65.46	65.44	+
12	65.48	66.29	–
13	64.90	65.72	–
14	65.10	63.13	+
15	65.24	64.79	+
16	65.25	65.25	0
17	65.50	65.21	+
18	65.61	64.10	+
19	65.52	65.77	–
20	64.62	64.83	–
21	64.14	65.25	–
22	65.14	65.21	–
23	65.06	65.94	–
24	65.12	65.05	+
25	66.06	64.66	+
26	66.42	65.34	+
27	62.86	64.46	–
28	64.54	64.09	+
29	64.28	65.16	–
30	63.36	64.14	–
31	65.32	64.54	+
32	63.62	63.73	–
33	64.53	65.17	–
34	65.48	65.44	+
35	65.48	66.29	–
36	65.10	63.13	+
37	64.74	65.11	–
38	65.03	64.25	+
39	65.71	65.65	+
40	64.90	65.72	–

8.7.1 *Null Hypothesis (H_0) and Alternative Hypothesis (H_1)*

The null hypothesis (H_0) is that there is no difference in estimation of iron content by the two sampling methods against an alternative hypothesis (H_1) that there is a difference between the two methods.

8.7.2 It can be seen that out of 40 differences, two are zero. Ignoring the zero's, from the remaining 38 differences, the number of positive signs is 16 and the negative signs is 22.

x = number of fewer signs (+ve) = 16

n = sample size = 38

p = (under H_0) = 1/2

$$Z = \frac{|2x-n|}{\sqrt{n}} = \frac{|32-38|}{\sqrt{38}} = 0.97$$

8.7.3 Since the calculated value is less than 1.96, it is concluded that the two methods of sampling are not significantly different at 5 percent level of significance.

Going by the p-value approach also, here since

$$Z = \frac{2x-n}{\sqrt{n}} = -0.97, \text{ and}$$

P $(Z < -0.97) = 0.166\,0$ is larger than 0.05, the same conclusion is reached.

8.8 Power Efficiency

The power efficiency of the sign test as compared to t-test is about 95 percent for sample size (n) = 6, but it declines as the sample size increases to an asymptotic efficiency of 63 percent.

9 WILCOXON MATCHED-PAIRS SIGN-RANK TEST

9.1 The sign test utilizes information simply about the direction of differences within pairs. But this test is based on the relative magnitude as well as the direction of the differences, that is, it gives more weight to a pair which shows a large difference than to a pair which shows a small difference.

9.2 For any matched pair, the difference between the two observations d is calculated. Such d's are ranked without regard to sign, that is, a rank of 1 is given to the smallest d, the rank of 2 to the next smallest and so on. Thus a difference of –1 will have a lower rank than a difference of either +2 or –2. Then the sign of the difference is assigned to each rank, that is, it is indicated as to which of the ranks are arising from the negative d's and which ranks are from positive d's.

9.3 If the difference between any pair is zero, that pair is dropped from the analysis and the sample size (n) is thereby reduced. It may also be possible that a tie may occur, that is, two or more pairs may have same numerical value of difference. The rank assigned in such cases is the average of the ranks which would have to be assigned if the d's had differed slightly. For example, three pairs may have the value of d as −1, −1 and +1. In this case each pair would be assigned the rank of 2, because the average of the ranks is $= \frac{1+2+3}{3} = 2$.

Then the next d in order would receive the rank of 4 because the ranks 1, 2, 3 have already been used.

9.4 Under the null hypothesis it is expected that the sum of the ranks having a plus sign and that having a minus sign should be equal. Therefore, if the sum of ranks of positive sign is very much different from that of negative sign, it is expected that there is a significant difference and the null hypothesis should be rejected.

9.5 Small Samples

This method shall be employed when the number of pairs (n) is less than or equal to 25. Let T be the smaller sum of like signed ranks, that is, T is either the sum of the positive ranks or the sum of the negative ranks, whichever is smaller. The value of T is calculated from a sample of n pairs and compared with the critical value for sample size n and desired level of significance. Depending upon the alternative hypothesis being two-sided or one-sided, the appropriate critical value may be chosen from Annex D. If the critical value is greater than the calculated value of T, then the null hypothesis is rejected, otherwise not. Alternatively, if the calculated p-value of the test-statistic assuming, under H_0, the observed value and more likely values favouring H_1, is less than the chosen level of significance α, H_0 is rejected.

9.6 Example 5

The yield of waxy substance from tobacco leaves is supposed to depend upon the solvent used for extraction. For each of the ten different sources from which leaves were obtained, two extractions, using solvent A and solvent B were carried out. The results are given in Table 7. It has to be tested whether the two solvents produce significantly different estimate of wax contents.

Table 7 Yield of Waxy Substance from Tobacco Leaves
(*Clause* 9.6)

Source (1)	Solvent A (2)	Solvent B (3)
1	2.3	3.0
2	3.2	2.7
3	2.5	2.8
4	4.8	4.3
5	4.2	5.2
6	2.8	4.0
7	3.6	3.6
8	4.6	3.2
9	3.9	4.8
10	4.5	5.8

9.6.1 *Null Hypothesis* (H_0) *and Alternative Hypothesis* (H_1)

The null hypothesis is that there is no difference between the estimates of wax content by two different solvents against an alternative hypothesis (H_1) that there is a significant difference between the estimates.

9.6.2 The difference d and the signed ranks of d are given in Table 8.

Table 8 Differences and Signed Ranks

Source (1)	Difference $D = A-B$ (2)	Rank of d (3)	Rank with Less Frequent Sign (4)
1	−0.7	−4	
2	0.5	2.5	2.5
3	−0.3	−1	
4	0.5	2.5	2.5
5	−1.0	−6	
6	−1.2	−7	
7	0	—	
8	1.4	9	9
9	−0.9	−5	
10	−1.3	−8	
	Total		14

9.6.3 Since for the source 7, the difference is zero, this pair is dropped from the analysis. So the sample size (n) will be reduced to 9. Since for the sources 2 and 4 the same difference 0.5 is obtained, their rank would be (2+3)/2 = 2.5 each. The sum of the positive and negative ranks is 14 and 31 respectively. The smaller of the values, that is, 14 is chosen as T. From Annex D, for a two-sided test, the critical value (for $n = 9$ and 5 percent level of significance) is 6. Since the calculated value is greater than the critical value, the null hypothesis that the two solvents do not produce significantly different estimate of wax content is not rejected.

Using a software package, the 2-tailed p-value comes out as $0.343 > 0.05$ so acceptance of H_0 is further confirmed.

9.7 Large Samples

When the samples size is more than 25, the smaller sum of the like signed ranks, T is approximately normally distributed with:

$$\text{Mean } (\bar{X}) = \frac{n(n+1)}{4} \text{ and,}$$

$$\text{Variance} = \frac{n(n+1)(2n+1)}{24}$$

$$\text{Therefore, } Z = \frac{\left|T - \frac{n(n+1)}{4}\right|}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}$$

is approximately normally distributed with mean zero and variance 1. The value of Z is calculated and compared with critical value of 1.96 (corresponding to 5 percent level of significance) or 2.58 (corresponding to 1 percent level of significance), for a two-sided test. For one-sided test, the calculated value is compared with critical value of 1.645 (corresponding to 5 percent level of significance) or 2.325 (corresponding to 1 percent level of significance). The null hypothesis is rejected if the calculated value of Z is greater than the critical value, otherwise not. Alternatively, if the calculated p-value of the test-statistic assuming, under H_0, the observed value and more likely values favouring H_1, is less than the chosen level of significance α, H_0 is rejected.

9.8 Example 6

Table 9 gives the rate of consumption of gasoline (km/l) by 30 motorcycles before and after the application of newly developed gasoline additives. It has to be examined whether there is any significant difference in the rate of consumption of gasoline after the application of the additive.

9.8.1 *Null Hypothesis (H_0) and Alternative Hypothesis (H_1)*

The null hypothesis is that there is no difference in the rate of consumption of gasoline after the application of the additive against an alternative hypothesis (H_1) that the additive reduces the rate of consumption (one-sided test). Let the distance travelled after application of additive (y) minus the distance travelled before application of additive (x) be denoted by d. Then under alternative hypothesis H_1, many of the values of d will be positive and therefore, the sum of positive ranks will be much larger than the sum of negative ranks.

Therefore, T = Smaller sum of like signed ranks = 29.5

$$Z = \frac{\left|29.5 - \frac{30 \times 31}{4}\right|}{\sqrt{\frac{30 \times 31 \times 61}{24}}} = 4.18$$

9.8.2 Since the calculated value of Z is greater than 1.645 (corresponding to 5 percent level of significance) or 2.325 (corresponding to 1 percent level of significance), the null hypothesis that there is no difference in the rate of consumption of gasoline after the application of additive is rejected at 1 percent level of significance.

Using the p-value approach, too, since $P(Z < -4.18) \simeq 0 < 0.01$, the same conclusion is upheld.

Table 9 Rate of Consumption of Gasoline (km/l)
(*Clause* 9.8)

Motor Cycle No.	Distance (km/l) Without Additive (x)	Distance (km/l) With Additive (y)	Difference $d = y - x$	Sign Rank
(1)	(2)	(3)	(4)	(5)
1	27.2	28.3	1.1	+15.5
2	31.6	30.8	–0.8	–8.5
3	29.8	30.9	1.6	+27
4	29.1	31.2	2.1	+30
5	32.0	32.7	0.9	+6
6	28.7	28.6	–0.1	–1
7	30.3	31.9	1.6	+27
8	28.3	28.9	0.6	+5
9	30.1	30.4	0.3	+3
10	27.8	28.9	1.1	+15.5
11	29.3	30.1	0.8	+8.5
12	30.4	32.0	1.6	+27
13	28.6	30.1	1.5	+24
14	29.5	30.4	0.9	+11.5
15	29.9	30.9	1.0	+13.5
16	28.2	29.1	0.9	+11.5
17	27.4	28.2	0.8	+8.5
18	27.5	27.7	0.2	+2
19	28.4	28.9	0.5	+4
20	27.8	29.1	1.3	+20
21	29.2	30.2	1.0	+13.5
22	29.9	31.1	1.2	+17.5
23	32.8	31.5	–1.3	–20
24	28.7	30.0	1.3	+20
25	30.8	31.6	0.8	+8.5
26	31.1	32.5	1.4	+22
27	27.8	29.3	1.5	+24
28	28.6	29.8	1.2	+17.5
29	30.2	31.9	1.7	+29
30	31.3	32.8	1.5	+24
			Total	+435.5
				–29.5

9.9 Power Efficiency

When all the assumptions of t-test are met, the power efficiency of Wilcoxon matched-pairs sign-rank test as compared to t-test is around 95 percent.

10 MEDIAN TEST

10.1 The median test is used to test whether two independent samples have been drawn from the populations with the same median. The null hypothesis (H_0) is that the two samples are from populations with the same median. The alternative hypothesis (H_1) may be that the median of one population is different from that of the other population (two-sided test).

10.2 Two samples of sizes n_1 and n_2 drawn from the two populations are pooled and the median is obtained for the combined sample of size $n_1 + n_2 = n$. The data is arranged in a 2 x 2 table as given in Table 10.

Table 10 2 x 2 Contingency Table

	Sample I	Sample II	Total
No. of observations equal to or greater than combined median	a	b	$a+b$
No. of observationsless than combined median	c	d	$c+d$
Total:	$n_1 = a+c$	$n_2 = b+d$	$n = n_1 + n_2$

10.3 If both the samples are drawn from the populations with the same median, it is expected that about half of each sample observations will be above the combined median and about half will be below, that is, it is expected that frequencies a and c would be almost same and so also frequencies b and d.

10.4 Large Samples

When the total number of observations in both the samples combined is more than 20, a χ^2-test for 2 x 2 contingency table as given below may be used for testing the null hypothesis:

$$\chi^2 = \frac{n\left[|ad-bc|-\frac{n}{2}\right]^2}{(a+b)(c+d)(b+d)(a+c)}$$

is distributed as χ^2 with one degree of freedom [*see also* IS 6200 (Part 2)].

10.4.1 The value of χ^2 for one degree of freedom is calculated and compared with critical value for 5 percent level or 1 percent level of significance for a two-sided test, as given below:

Critical Values for Upper Tail and Lower Tail with Equal Area

Significance Level, α	*Lower Tail*	*Upper Tail*
0.05	0.000 98	5.02
0.01	0.000 039	7.88

The null hypothesis is rejected if the calculated value is greater than the critical value of upper tail or smaller than the critical value of the lower tail, otherwise it is not rejected.

10.5 Example 7

In a factory, two machines were used for manufacturing steel tubes of nominal outside diameter 60 mm. Table 11 gives the outside diameter of steel tubes randomly selected from both the machines. Test whether the two machines are manufacturing the tubes of same diameter.

Table 11 Outside Diameter of Steel Tubes, mm

Sl No.	Machine I	Combined Rank	Sl No.	Machine II	Combined Rank
1	59.2	1	1	59.4	4.5
2	60.2	18.5	2	59.5	6
3	59.3	2.5	3	60.7	27
4	60.5	24.5	4	59.8	12.5
5	60.5	24.5	5	59.8	12.5
6	60.6	26	6	59.8	12.5
7	60.3	20.5	7	59.7	9
8	59.3	2.5	8	60.0	15.5
9	59.6	7	9	59.7	9
10	60.3	20.5	10	59.4	4.5
11	60.4	22.5	11	60.1	17
12	59.7	9	12	60.0	15.5
13	60.4	22.5			
14	60.2	18.5			
15	59.8	12.5			

10.5.1 The values from both samples when combined and arranged in ascending order, the combined median comes out as 59.8. The data is arranged in 2 x 2 contingency table, as shown in Table 12.

Table 12 2 x 2 Contingency Table

	Machine I	Machine II	Total
No. of observations greater than or equal to combined median	9 (c)	4 (d)	13
No. of observations less than combined median	6 (a)	8 (b)	14
Total :	$n_1 = 15$	$n_2 = 12$	$n = 27$

The calculation for χ^2 may also be shown as:

$$\chi^2 = \frac{n\left[|ad-bc|-n/2\right]^2}{(a+b)(c+d)(b+d)(a+c)}$$

$$= \frac{\left[27(24-72)-27/2\right]^2}{14\times13\times12\times15}$$

$$= \frac{27\times(34.5)^2}{32\ 760} = 0.981$$

10.5.2 Since the calculated value of χ^2 is less than 3.84, the tabulated value at 5 percent level of significance, the null hypothesis that both the machines are manufacturing tubes of same diameter is not rejected.

Using a software package, the p-value comes out as $0.256 > 0.05$, so that acceptance of H_0 is upheld.

10.6 Power Efficiency

The power efficiency of a median test as compared to t-test is about 95 percent for $n_1 + n_2 = 6$. The power efficiency decreases as the sample size increases, reaching an asymptotic efficiency of 63 percent.

11 MANN-WHITNEY *U* TEST

11.1 The Mann-Whitney U test is used to test whether two independent samples have been drawn from the same population. This is one of the most powerful non-parametric tests and is most useful alternative to the parametric t-test.

11.2 Suppose two samples are drawn from two populations A and B. The null hypothesis is that both the populations are identical. The alternative hypothesis may be that the location parameter of one population is larger (or smaller) than the location parameter of the other population, that is, the bulk of the distribution of one population is to the right (or to left) of the bulk of the distribution of the other (one-sided test). For a two-sided test, the alternative hypothesis is that the two are not identical.

11.3 To apply this test, let n_1 = the number of observations in the smaller of two independent samples, and n_2 = the number of observations in the larger of two independent samples.

11.3.1 The observations from both the samples are combined and arranged in non-descending order with the identity of the samples preserved. The ranks are given in order of increasing size. In this ranking algebraic size is considered, that is, the lowest rank is assigned to the largest negative number if any. In case a tie occurs, each of the tied observations is given the average of ranks which they would have had if the values had differed slightly. Calculate the sum of ranks assigned to a sample with n_1 observations (say R_1). Similarly find the sum of ranks assigned to a sample with n_2 observations (say R_2). Two values U_1 and U_2 are calculated by the following relations:

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - R_2$$

NOTES

1 In fact U_1 is the number of times that an observation in a sample of size n_2 precedes an observation in a sample of size n_1. Similarly U_2 may be defined.

2 The values of U_1 and U_2 as calculated above may also be verified by the following relation:

$$U_1 + U_2 = n_1 n_2$$

11.3.2 The smaller of U_1 and U_2 is taken as the value of U.

11.4 Small Samples

The following method shall be employed when n_2, the number of observations in the larger of two independent samples is less than or equal to 20.

11.4.1 The value of U as calculated in **11.3.2** is compared with the critical value for a given n_1, n_2 and desired level of significance. The critical values for this purpose are given in Annexes E and F (for one-sided test) and Annexes G and H (for two-sided test). The null hypothesis shall be rejected if the calculated value of U is less than the critical value, otherwise not. Alternatively, if the calculated p-value of the test-statistic assuming, under H_0, the observed value and more likely values favouring H_1, is less than the chosen level of significance α, H_0 is rejected.

11.5 Example 8

Tests were conducted by a consumer organization on two types of flares, Super-flash and Britalite. Table 13 gives burning time (in min) for 12 flares of each make. It has to be tested whether Super-flash has more burning time. Also compare its conclusion with parametric t-test.

Table 13 Burning Time, min

(*Clauses* 11.5 *and* 11.5.4)

Sl No. (1)	Super-Flash A (2)	Combined Rank (3)	Britalite B (4)	Combined Rank (5)
1	20.9	19	15.9	10
2	19.3	17	15.5	6
3	19.6	18	17.4	15
4	23.3	23	18.0	16
5	21.2	20	13.9	3
6	22.4	22	15.6	7
7	14.2	4	15.8	9
8	16.5	12	13.4	2
9	16.7	13	10.1	1
10	17.3	14	24.3	24
11	15.2	5	15.7	8
12	21.4	21	16.4	11

11.5.1 *Null Hypothesis (H_0) and Alternative Hypothesis (H_1)*

The null hypothesis is that the two types of flares do not differ in the burning time against an alternative hypothesis (H_1) that Super-flash has more burning time.

11.5.2 The sequence of observations of the two samples when combined and arranged in non-descending order is given by:

	B	B	B	A	A	B	B	B	B	B	B	A
Ranks	1	2	3	4	5	6	7	8	9	10	11	12
	A	A	B	B	A	A	A	A	A	A	A	B
Ranks	13	14	15	16	17	18	19	20	21	22	23	24

where A denotes an observation from Super-flash and B denotes an observation from Britalite.

Here R_1 = sum of ranks of observations from Super-flash = 188

R_2 = sum of ranks of observations from Britalite = 112

$$U_1 = 144 + \frac{12 \times 13}{2} - 188$$

$$= 222 - 188 = 34$$

$$\text{Similarly, } U_2 = 144 + \frac{12 \times 13}{2} - 112$$

$$= 222 - 112 = 110$$

Therefore, U = Minimum of U_1 and U_2 = 34

11.5.3 The critical value (one-sided test) for n_1 = 12, n_2 = 12 and 5 percent level of significance is 42 (*see* Annex E). Since the calculated value of U is less than the critical value, the null hypothesis that there is no difference in the burning time for both types of flashes is rejected.

11.5.4 Applying t-test to the data given in Table 13, we get:

for Super-flash n_1 = 12, mean $(\bar{x})$ = 19, and

for Britalite n_2 = 12 and mean $(\bar{y})$ = 16

$$\text{also } S^2 = \frac{\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2}{n_1 + n_2 - 2} = 9.92$$

$$\text{Therefore, } t = \frac{(\bar{x}-\bar{y})}{S\left[1/n_1 + 1/n_2\right]^{1/2}} = \frac{(19-16)(6)^{1/2}}{3.15} = 2.33$$

The critical value (one-sided) of t for 5 percent level of significance and 22 degrees of freedom is 1.717. As the calculated value is greater than the critical value, the null hypothesis is rejected, thereby arriving at the same conclusion as by Mann-Whitney U test.

11.6 Large Samples (n_1, n_2 Larger than 20)

As n_1 and n_2 increase in size, the distribution of U approaches the normal with:

$$\text{Mean } (\bar{x}) = \frac{n_1 \, n_2}{2}, \text{ and}$$

$$\text{Variance} = \frac{n_1 \, n_2 (n_1 + n_2 + 1)}{12}$$

$$\text{Therefore, } Z = \frac{\dfrac{\left|U - n_1 \, n_2\right|}{2}}{\sqrt{\dfrac{n_1 \, n_2 (n_1 + n_2 + 1)}{12}}}$$

is standardized normal variate. The value of Z is calculated and compared with the critical value as given in **8.6.1**. Alternatively, if the calculated p-value of the test-statistic assuming, under H_0, the observed value and more likely values favouring H_1, is less than the chosen level of significance α, H_0 is rejected.

11.7 Example 9

In an experiment, 45 mentally retarded sub-normal patients with behaviour disorders were randomly divided into two groups of sizes 22 and 23 respectively. Those, in Group B were given inert tablets whereas those in Group A were treated with a tranquillizer. At the end of the period of treatment, all the patients were rated on the Claridge exitability rating scale, on which the highest score corresponds to the most distinguished behaviour. It is desired to test whether the tranquillizer is more effective in improving the patient's behaviour. The scores are as follows:

Group A	Combined Rank	Sl No.	Sample B Rank	Combined
84	14	1	82	12
141	31	2	70	6
224	44	3	76	10
72	8	4	118	24
154	33.5	5	100	20
218	43	6	174	36
91	17	7	135	28
137	30	8	88	16
209	41	9	78	11
111	21	10	128	26
238	45	11	74	9
147	32	12	58	4
193	40	13	135	28
96	18	14	185	39
154	33.5	15	46	3
210	42	16	41	1
119	25	17	71	7
178	37	18	135	28
182	38	19	116	23
160	35	20	83	13
99	19	21	69	5
114	22	22	86	15
	23	44	2	

11.7.1 *Null Hypothesis (H_0) and Alternative Hypothesis (H_1)*

The null hypothesis is that both the treatments are equally effective against an alternative hypothesis (H_1)

that the tranquillizer is more effective in improving the patient's behaviour.

11.7.2 The sequence of the observations of the two samples, when combined and arranged in the ascending order, the following sum of ranks were obtained:

R_1 = sum of ranks of observations from Group A = 669

R_2 = sum of ranks of observations from Group B = 366

Also n_1 = 22 and n_2 = 23

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - R_1$$

$$= 506 + 253 - 669 = 90$$

and $U_2 = n_1 n_2 - U_1 = 416$

Therefore, U = Minimum $(U_1, U_2) = 90$

$$\text{Mean} = \frac{n_1 n_2}{2} = 253$$

$$\text{Variance} = \frac{n_1 n_2 (n_1 + n_2 + 1)}{12} = 1\ 939.67$$

$$\text{Standard deviation} = \sqrt{1\ 939.67} = 44.04$$

$$Z = \frac{|90 - 253|}{44.04} = 3.70$$

Since the calculated value of Z is greater than 2.325, the null hypothesis is rejected thereby implying that the tranquillizer is more effective in improving patient's behaviour at 1 percent level of significance.

The conclusion is upheld by the p-value approach as well, since $P(Z < -3.70) \simeq 0 < 0.01$.

11.8 Power Efficiency

The power efficiency of Mann-Whitney U test as compared to t-test is about 95.5 percent.

12 WALD-WOLFOWITZ RUN TEST

12.1 The Wald-Wolfowitz run test is used to test the null hypothesis that the two independent samples have been drawn from the same population against an alternative hypothesis that the two populations differ in any respect whatsoever. The two populations may differ in central tendency, variability, skewness, etc. Thus this test may be used to test a large number of alternative hypothesis whereas many other tests are applicable to a particular type of difference between the two populations (for example, the median test determines whether the two samples have been drawn from two populations with the same median).

12.2 Two samples of sizes n_1 and n_2 drawn from two populations are pooled and the observations of both the samples are arranged in increasing order. The total number of runs is determined. A run is defined as any sequence of observations from the same sample. For example, the following order of sequence of size $n_1 + n_2 = 10$ may be observed:

A B A A B B B A B B

r = Number of runs of A + number of runs of B = 3 + 3 = 6

12.2.1 The number of runs may also be calculated by noting down the number of transitions from A to B or from B to A and using the following relation:

Total number of runs (r) = Number of transitions + 1

12.3 It is also possible that ties may occur. If ties are within the same sample, then there is no problem, as the number of runs is not affected. But if there be ties among observations from both the samples, one may not get a unique value of r. In that case, one has to break ties in all possible ways and find the corresponding values of r. If all these different r's lead to the same conclusion at the desired level of significance, there is no problem. In case different values of r lead to different decisions, we accept the largest among these values as a conservative approach (that is, an approach that rejects H_0 rather cautiously).

X sample : 17, 18, 19, 19

Y sample : 16, 19, 19

Combined Sequence		*16*	*17*	*18*	*19*	*19*	*19*	*19*	*Value of* r
Different ways of breaking ties	I	Y	X	X	X	X	Y	Y	3
	II	Y	X	X	X	Y	X	Y	5
No. of ways $= \frac{4!}{2!\,2!} = 6$	III	Y	X	X	X	Y	Y	X	4
	IV	Y	X	X	Y	X	Y	X	6
	V	Y	X	X	Y	X	X	Y	5
	VI	Y	X	X	Y	Y	X	X	4

Here we take $r = 6$

However, if the number of ties across the samples is large, the run test is not recommended.

12.4 If the two samples are drawn from the same population, that is, if H_0 is true, then the observations of A's and the B's will be well mixed. In that case r, the number of runs will be relatively large. Therefore if H_0 is true, the value of r will be large and if H_0 is false, the value of r will be small.

12.5 Small Samples

This method shall be employed when n_1 and n_2 is less than or equal to 20. The number of runs (r) is calculated by the method given in **12.2** and this value of r is compared with the critical value of r for a given n_1, n_2

and desired level of significance. The critical values of r are given in Annex J for 5 percent level of significance and in Annex K for 1 percent level of significance. The null hypothesis is rejected if the calculated value of r is less than the critical value, otherwise not. Alternatively, if the calculated p-value of the test-statistic is less than the chosen level of significance α, H_0 is rejected.

12.6 Example 10

The members of consumer association investigated two brands of canned peas selling at the same price for the same size of can. A random selection of 5 cans of brand A and 7 cans of brand B were made. The drained weight (in g) of the cans is given in Table 14. It is desired to test the hypothesis whether both the brands are equally good with regard to the net content.

Table 14 Drained Weights of Cans, g

Brand A	Brand B
297	280
292	308
312	311
307	293
317	314
	316
	296

12.6.1 *Null Hypothesis (H_0) and Alternative Hypothesis (H_1)*

The null hypothesis is that both the brands are equally good with regard to the net content.

12.6.2 The observations from both the brands when pooled and arranged in ascending order, the following sequence is obtained:

B A B B A A B B A B B A

So, r = Number of runs = 8

12.6.3 The critical value of r for $n_1 = 5$, $n_2 = 7$ and 5 percent level of significance is 3. Since the calculated value of r is greater than the critical value, the null hypothesis is not rejected.

12.6.4 *Comparison with* t-*test*

For brand A:

$$\text{Mean } (\bar{x}) = 305.0$$

$$\text{Variance } (S_1^2) = 86.00$$

For brand B:

$$\text{Mean } (\bar{y}) = 302.6$$

$$\text{Variance } (S_2^2) = 150.82$$

$$S^2 = \frac{\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2}{n_1+n_2-2}$$

$$= \frac{n_1 S_1^2 + n_2 S_2^2}{n_1+n_2-2}$$

$$S^2 = 148.57$$

$$S = 12.19$$

$$\text{Therefore, } t = \frac{|\bar{x}-\bar{y}|}{S[1/n_1 + 1/n_2]^{1/2}} = \frac{2.4}{12.19} \times \left[\frac{35}{12}\right]^{1/2} = 0.34$$

12.6.4.1 The tabulated value of t at 5 percent level of significance (two-sided test) is 2.23. Since the calculated value is less than the tabulated value, the null hypothesis that there is no significance difference between the two brands is not rejected, thereby leading to the same conclusion as by Wald Wolfowitz-run test.

12.7 Large Samples

When either n_1 or n_2 is greater than 20, then the number of runs is approximately normally distributed with:

$$\text{Mean } (\bar{\mu}_r) = \frac{2n_1n_2}{n_1+n_2} + 1, \text{ and}$$

$$\text{Variance } (\sigma_r^2) = \frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1+n_2)^2(n_1+n_2-1)}$$

$$\text{Therefore, } Z = \frac{|r-\bar{\mu}_r|}{\sigma_r} \text{ is a standardized normal variate.}$$

12.7.1 When $n_1 + n_2$ is less than 30 (with either n_1 *or* n_2 more than 20), the continuity correction should be applied in Z by subtracting 0.5 from the absolute difference of $(r - \mu_r)$. Thus in this case value of Z will be given by:

$$Z = \frac{|r-\bar{\mu}_r| - 0.5}{\sigma_r}$$

12.7.2 The value of Z is calculated and compared with the critical value as given in **8.6.1**. The null hypothesis is rejected if the calculated value of Z is greater than the critical value, otherwise not. Alternatively, if the calculated p-value of the test-statistic assuming, under H_0, the observed value and more likely values favouring H_1, is less than the chosen level of significance α, H_0 is rejected.

ANNEX A

(*Clause* 6.3)

CRITICAL VALUES OF *D* IN THE KOLMOGOROV-SMIRNOV ONE-SAMPLE TEST

Sample Size (n)	*Level of Significance*	
	5 Percent	1 Percent
1	0.975	0.995
2	0.842	0.929
3	0.708	0.828
4	0.624	0.733
5	0.565	0.669
6	0.521	0.618
7	0.486	0.577
8	0.457	0.543
9	0.432	0.514
10	0.410	0.490
11	0.391	0.468
12	0.375	0.450
13	0.361	0.433
14	0.349	0.418
15	0.338	0.404
16	0.328	0.392
17	0.318	0.381
18	0.309	0.371
19	0.301	0.363
20	0.294	0.356
25	0.27	0.32
30	0.24	0.29
35	0.23	0.27
Over 35	$1.36 / \sqrt{n}$	$1.63 / \sqrt{n}$

ANNEX B

(*Clauses* 7.7 *and* 7.8.3)

CRITICAL VALUES OF K_D IN THE KOLMOGOROV-SMIRNOV TWO-SAMPLE TEST

Sample Size (n)	*One-Sided Test for Level of Significance*		*Two-Sided Test for Level of Significance*	
	5 Percent	1 Percent	5 Percent	1 Percent
3	3	–	–	–
4	4	–	4	–
5	4	5	5	5
6	5	6	5	6
7	5	6	6	6
8	5	6	6	7
9	6	7	6	7
10	6	7	7	8
11	6	8	7	8
12	6	8	7	8
13	7	8	7	9
14	7	8	8	9
15	7	9	8	9
16	7	9	8	10
17	8	9	8	10
18	8	10	9	10
19	8	10	9	10
20	8	10	9	11
21	8	10	9	11
22	9	11	9	11
23	9	11	10	11
24	9	11	10	12
25	9	11	10	12
26	9	11	10	12
27	9	12	10	12
28	10	12	11	13
29	10	12	11	13
30	10	12	11	13
35	11	13	12	14
40	11	14	13	15

ANNEX C

(*Clauses* 8.4 *and* 8.5.2)

CRITICAL VALUES OF *X* IN THE SIGN TEST

Sample Size (n)	*One-Sided Test for Level of Significance*		*Two-Sided Test for Level of Significance*	
	5 Percent	1 Percent	5 Percent	1 Percent
5	0	–	–	–
6	0	–	0	–
7	0	0	0	–
8	1	0	0	0
9	1	0	1	0
10	1	0	1	0
11	2	1	1	0
12	2	1	2	1
13	3	1	2	1
14	3	2	2	1
15	3	2	3	2
16	4	3	3	2
17	4	3	4	2
18	5	3	4	3
19	5	4	4	3
20	5	4	5	3
21	6	4	5	4
22	6	5	5	4
23	7	5	6	4
24	7	5	6	5
25	7	6	7	5

ANNEX D

(*Clauses* 9.5 *and* 9.6.3)

CRITICAL VALUES OF *T* IN THE WILCOXON MATCHED-PAIRS SIGN-RANK TEST

Sample Size (*n*)	*One-Sided Test for Level of Significance*		*Two-Sided Test for Level of Significance*	
	5 Percent	1 Percent	5 Percent	1 Percent
6	2	—	0	—
7	3	0	2	—
8	5	2	4	0
9	8	3	6	2
10	10	5	8	3
11	13	7	11	5
12	17	10	14	7
13	21	13	17	10
14	25	16	21	13
15	30	20	25	16
16	35	24	30	20
17	41	28	35	23
18	47	33	40	28
19	53	38	46	32
20	60	43	52	38
21	67	49	59	43
22	75	56	66	49
23	83	62	73	55
24	91	69	81	61
25	100	77	89	68

ANNEX E

(*Clauses* 11.4.1 *and* 11.5.3)

CRITICAL VALUES OF *U* IN MANN-WHITNEY *U* TEST (ONE-SIDED) FOR 5 PERCENT LEVEL OF SIGNIFICANCE

n_1/n_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0	0
2	–	–	–	–	0	0	0	1	1	1	1	2	2	2	3	3	3	4	4	4
3	–	–	0	0	1	2	2	3	3	4	5	5	6	7	7	8	9	9	10	11
4	–	–	0	1	2	3	4	5	6	7	8	9	10	11	12	14	15	16	17	18
5	–	0	1	2	4	5	6	8	9	11	12	13	15	16	18	19	20	22	23	25
6	–	0	2	3	5	7	8	10	12	14	16	17	19	21	23	25	26	28	30	32
7	–	0	2	4	6	8	11	13	15	17	19	21	24	26	28	30	33	35	37	39
8	–	1	3	5	8	10	13	15	18	20	23	26	28	31	33	36	39	41	44	47
9	–	1	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54
10	–	1	4	7	11	14	17	20	24	27	31	34	37	41	44	48	51	55	58	62
11	–	1	5	8	12	16	19	23	27	31	34	38	42	46	50	54	57	61	65	69
12	–	2	5	9	13	17	21	26	30	34	38	42	47	51	55	60	64	68	72	77
13	–	2	6	10	15	19	24	28	33	37	42	47	51	56	61	65	70	75	80	84
14	–	2	7	11	16	21	26	31	36	41	46	51	56	61	66	71	77	82	87	92
15	–	3	7	12	18	23	28	33	39	44	50	55	61	66	72	77	83	88	94	100
16	–	3	8	14	19	25	30	36	42	48	54	60	65	71	77	83	89	95	101	107
17	–	3	9	15	20	26	33	39	45	51	57	64	70	77	83	89	96	102	109	115
18	–	4	9	16	22	28	35	41	48	55	61	68	75	82	88	95	102	109	116	123
19	0	4	10	17	23	30	37	44	51	58	65	72	80	87	94	101	109	116	123	130
20	0	4	11	18	25	32	39	47	54	62	69	77	84	92	100	107	115	123	130	138

ANNEX F

(*Clause* 11.4.1)

CRITICAL VALUES OF *U* IN MANN-WHITNEY *U*-TEST (ONE-SIDED) FOR 1 PERCENT LEVEL OF SIGNIFICANCE

n_1/ n_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
2	–	–	–	–	–	–	–	–	–	–	–	–	0	0	0	0	0	0	1	1
3	–	–	–	–	–	–	0	0	1	1	1	2	2	2	3	3	4	4	4	5
4	–	–	–	–	0	1	1	2	3	3	4	5	5	6	7	7	8	9	9	10
5	–	–	–	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
6	–	–	–	1	2	3	4	6	7	8	9	11	12	13	15	16	18	19	20	22
7	–	–	0	1	3	4	6	7	9	11	12	14	16	17	19	21	23	24	26	28
8	–	–	0	2	4	6	7	9	11	13	15	17	20	22	24	26	28	30	32	34
9	–	–	1	3	5	7	9	11	14	16	18	21	23	26	28	31	33	36	38	40
10	–	–	1	3	6	8	11	13	16	19	22	24	27	30	33	36	38	41	44	47
11	–	–	1	4	7	9	12	15	18	22	25	28	31	34	37	41	44	47	50	53
12	–	–	2	5	8	11	14	17	21	24	28	31	35	38	42	46	49	53	56	60
13	–	0	2	5	9	12	16	20	23	27	31	35	39	43	47	51	55	59	63	67
14	–	0	2	6	10	13	17	22	26	30	34	38	43	47	51	56	60	65	69	73
15	–	0	3	7	11	15	19	24	28	33	37	42	47	51	56	61	66	70	75	80
16	–	0	3	7	12	16	21	26	31	36	41	46	51	56	61	66	71	76	82	87
17	–	0	4	8	13	18	23	28	33	38	44	49	55	60	66	71	77	82	88	93
18	–	0	4	9	14	19	24	30	36	41	47	53	59	65	70	76	82	88	94	100
19	–	1	4	9	15	20	26	32	38	44	50	56	63	69	75	82	88	94	101	107
20	–	1	5	10	16	22	28	34	40	47	53	60	67	73	80	87	93	100	107	114

ANNEX G

(*Clause* 11.4.1)

CRITICAL VALUES OF *U* IN MANN-WHITNEY *U*-TEST (TWO-SIDED) FOR 5 PERCENT LEVEL OF SIGNIFICANCE

n_1/ n_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
2	–	–	–	–	–	–	–	0	0	0	0	1	1	1	1	1	2	2	2	2
3	–	–	–	–	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8
4	–	–	–	0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	13
5	–	–	0	1	2	3	5	6	7	8	9	11	12	13	14	15	17	18	19	20
6	–	–	1	2	3	5	6	8	10	11	13	14	16	17	19	21	22	24	25	27
7	–	–	1	3	5	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34
8	–	0	2	4	6	8	10	13	15	17	19	22	24	26	29	31	34	36	38	41
9	–	0	2	4	7	10	12	15	17	20	23	26	28	31	34	37	39	42	45	48
10	–	0	3	5	8	11	14	17	20	23	26	29	33	36	39	42	45	48	52	55
11	–	0	3	6	9	13	16	19	23	26	30	33	37	40	44	47	51	55	58	62
12	–	1	4	7	11	14	18	22	26	29	33	37	41	45	49	53	57	61	65	69
13	–	1	4	8	12	16	20	24	28	33	37	41	45	50	54	59	63	67	72	76
14	–	1	5	9	13	17	22	26	31	36	40	45	50	55	59	64	67	74	78	83
15	–	1	5	10	14	19	24	29	34	39	44	49	54	59	64	70	75	80	85	90
16	–	1	6	11	15	21	26	31	37	42	47	53	59	64	70	75	81	86	92	98
17	–	2	6	11	17	22	28	34	39	45	51	57	63	67	75	81	87	93	99	105
18	–	2	7	12	18	24	30	36	42	48	55	61	67	74	80	86	93	99	106	112
19	–	2	7	13	19	25	32	38	45	52	58	65	72	78	85	92	99	106	113	119
20	–	2	8	13	20	27	34	41	48	55	62	69	76	83	90	98	105	112	119	127

ANNEX H

(*Clause* 11.4.1)

CRITICAL VALUES OF *U* IN MANN-WHITNEY *U* TEST (TWO-SIDED) FOR 1 PERCENT LEVEL OF SIGNIFICANCE

n_1/n_2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0	0
3	–	–	–	–	–	–	–	–	0	0	0	1	1	1	2	2	2	2	3	3
4	–	–	–	–	–	0	0	1	1	2	2	3	3	4	5	5	6	6	7	8
5	–	–	–	–	0	1	1	2	3	4	5	6	7	7	8	9	10	11	12	13
6	–	–	–	0	1	2	3	4	5	6	7	9	10	11	12	13	15	16	17	18
7	–	–	–	0	1	3	4	6	7	9	10	12	13	15	16	18	19	21	22	24
8	–	–	–	1	2	4	6	7	9	11	13	15	17	18	20	22	24	26	28	30
9	–	–	0	1	3	5	7	9	11	13	16	18	20	22	24	27	29	31	33	36
10	–	–	0	2	4	6	9	11	13	16	18	21	24	26	29	31	34	37	39	42
11	–	–	0	2	5	7	10	13	16	18	21	24	27	30	33	36	39	42	45	48
12	–	–	1	3	6	9	12	15	18	21	24	27	31	34	37	41	44	47	51	54
13	–	–	1	3	7	10	13	17	20	24	27	31	34	38	42	45	49	53	57	60
14	–	–	1	4	7	11	15	18	22	26	30	34	38	42	46	50	54	58	63	67
15	–	–	2	5	8	12	16	20	24	29	33	37	42	46	51	55	60	64	69	73
16	–	–	2	5	9	13	18	22	27	31	36	41	45	50	55	60	65	70	74	79
17	–	–	2	6	10	15	19	24	29	34	39	44	49	54	60	65	70	75	81	86
18	–	–	2	6	11	16	21	26	31	37	42	47	53	58	64	70	75	81	87	92
19	–	0	3	7	12	17	22	28	33	39	45	51	57	63	69	74	81	87	93	99
20	–	0	3	8	13	18	24	30	36	42	48	54	60	67	73	79	86	92	99	105

ANNEX J

(*Clause* 12.5)

CRITICAL VALUES OF *r* IN WALD-WOLFOWITZ RUN TEST FOR 5 PERCENT LEVEL OF SIGNIFICANCE

n_1/ n_2	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	–	–	–	–	–	–	–	–	–	–	2	2	2	2	2	2	2	2	2
3	–	–	–	–	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3
4	–	–	–	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4	4
5	–	–	2	2	3	3	3	3	3	4	4	4	4	4	4	4	5	5	5
6	–	2	2	3	3	3	3	4	4	4	4	5	5	5	5	5	5	6	6
7	–	2	2	3	3	3	4	4	5	5	5	5	5	6	6	6	6	6	6
8	–	2	3	3	3	4	4	5	5	5	6	6	6	6	6	7	7	7	7
9	–	2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8	8
10	–	2	3	3	4	5	5	5	6	6	7	7	7	7	8	8	8	8	9
11	–	2	3	4	4	5	5	6	6	7	7	7	8	8	8	9	9	9	9
12	2	2	3	4	4	5	6	6	7	7	7	8	8	8	9	9	9	10	10
13	2	2	3	4	5	5	6	6	7	7	8	8	9	9	9	10	10	10	10
14	2	2	3	4	5	5	6	7	7	8	8	9	9	9	10	10	10	11	11
15	2	3	3	4	5	6	6	7	7	8	8	9	9	10	10	11	11	11	12
16	2	3	4	4	5	6	6	7	8	8	9	9	10	10	11	11	11	12	12
17	2	3	4	4	5	6	7	7	8	9	9	10	10	11	11	11	12	12	13
18	2	3	4	5	5	6	7	8	8	9	9	10	10	11	11	12	12	13	13
19	2	3	4	5	6	6	7	8	8	9	10	10	11	11	12	12	13	13	13
20	2	3	4	5	6	6	7	8	9	9	10	10	11	12	12	13	13	13	14

ANNEX K

(*Clause* 12.5)

CRITICAL VALUES OF *r* IN WALD-WOLFOWITZ RUN TEST FOR 1 PERCENT LEVEL OF SIGNIFICANCE

n_1/ n_2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
3	–	–	–	–	–	–	–	–	–	2	2	2	2	2	2	2	2	2
4	–	–	–	–	–	2	2	2	2	2	2	2	3	3	3	3	3	3
5	–	–	–	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4
6	–	–	2	2	2	3	3	3	3	3	3	4	4	4	4	4	4	4
7	–	–	2	2	3	3	3	3	4	4	4	4	4	5	5	5	5	5
8	–	2	2	3	3	3	3	4	4	4	5	5	5	5	5	6	6	6
9	–	2	2	3	3	3	4	4	5	5	5	5	6	6	6	6	6	7
10	–	2	3	3	3	4	4	5	5	5	5	6	6	6	7	7	7	7
11	–	2	3	3	4	4	5	5	5	6	6	6	7	7	7	7	8	8
12	2	2	3	3	4	4	5	5	6	6	6	7	7	7	8	8	8	8
13	2	2	3	3	4	5	5	5	6	6	7	7	7	8	8	8	9	9
14	2	2	3	4	4	5	5	6	6	7	7	7	8	8	8	9	9	9
15	2	3	3	4	4	5	6	6	7	7	7	8	8	9	9	9	10	10
16	2	3	3	4	5	5	6	6	7	7	8	8	9	9	9	10	10	10
17	2	3	3	4	5	5	6	7	7	8	8	8	9	9	10	10	10	11
18	2	3	4	4	5	6	6	7	7	8	8	9	9	10	10	11	11	11
19	2	3	4	4	5	6	6	7	8	8	9	9	10	10	10	11	11	12
20	2	3	4	4	5	6	7	7	8	8	9	9	10	10	11	11	12	12

Bureau of Indian Standards

BIS is a statutory institution established under the *Bureau of Indian Standards Act*, 1986 to promote harmonious development of the activities of standardization, marking and quality certification of goods and attending to connected matters in the country.

Review of Indian Standards

Amendments are issued to standards as the need arises on the basis of comments. Standards are also reviewed periodically; a standard along with amendments is reaffirmed when such review indicates that no changes are needed; if the review indicates that changes are needed, it is taken up for revision. Users of Indian Standards should ascertain that they are in possession of the latest amendments or edition by referring to the latest issue of 'BIS Catalogue' and 'Standards : Monthly Additions'.

This Indian Standard has been developed from Doc : No. MSD 3 (323).

Amendments Issued Since Publication

Amend No.	Date of Issue	Text Affected

BUREAU OF INDIAN STANDARDS

Headquarters :

Manak Bhavan, 9 Bahadur Shah Zafar Marg, New Delhi 110 002
Telephones : 2323 0131, 2323 3375, 2323 9402

Telegrams : Manaksanstha
(Common to all offices)

Regional Offices :		Telephone
Central	: Manak Bhavan, 9 Bahadur Shah Zafar Marg NEW DELHI 110 002	2323 7617 2323 3841
Eastern	: 1/14 C.I.T. Scheme VII M, V. I. P. Road, Kankurgachi KOLKATA 700 054	2337 8499, 2337 8561 2337 8626, 2337 9120
Northern	: SCO 335-336, Sector 34-A, CHANDIGARH 160 022	260 3843 260 9285
Southern	: C.I.T. Campus, IV Cross Road, CHENNAI 600 113	2254 1216, 2254 1442 2254 2519, 2254 2315
Western	: Manakalaya, E9 MIDC, Marol, Andheri (East) MUMBAI 400 093	2832 9295, 2832 7858 2832 7891, 2832 7892

Branches : AHMEDABAD. BANGALORE. BHOPAL. BHUBANESHWAR. COIMBATORE. FARIDABAD. GHAZIABAD. GUWAHATI. HYDERABAD. JAIPUR. KANPUR. LUCKNOW. NAGPUR. PARWANOO. PATNA. PUNE. RAJKOT. THIRUVANANTHAPURAM. VISAKHAPATNAM.

Printed by Sunshine Graphics